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# Triple exponential smoothing; a tool for common stock price prediction

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TRIPLE EXPONENTIAL SMOOTHING

A TOOL FOR COMMON STOCK PRICE PREDICTION

By

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## CHAPTER I

## INTRODUCTION

What factors are important and relevant in determining the market value of common stocks? If obtaining an answer to this question was such an easy task, then many countless hours and dollars have been spent in vain, by both individuals and vast investment institutions. As stock prices are subject to the influence of many variables, it is not surprising to find a multitude of approaches to stock evaluation. Dollar-cost averaging, constant and variable stock-bond ratio plans, trend and business-cycle analysis, the low-price earnings out-of-favor approach, and resistance points or the price and volume formations analysis are but a few of the numerous investment strategies that have been used by investors for common stock selection and investment strategy timing.

Many investment people on Wall Street believe that the capitalization of earnings is the most important basis for investment decisions relative to common stocks. The quality, or volatility, and the growth rate of earnings determine the capitalization rate and the price-earnings ratio that investors are presently willing to pay for a stock. This current appraisal of the company, in relation to all other companies, by all interested buyers and sellers, determines the market value of the stock at any given point in time. As our economy is dynamic and changing, these appraisals vary, causing the price of any given stock to change. It is the present evaluation of a company that is of significance and to which major consideration should be given.

Although many investors believe that price should be the primary criterion for stock selection and that substantial capital gains can be made by investing in major companies within major industries that have withstood the tests of competition; other investors believe that a company's dividend policy should be of primary concern. The utilities industry is a classic example of the effect that dividend policies can have on the price of a stock. Many investors, due to age, income-tax status, or aversion to risk, believe that dividends should be the major factor for common stock selection. This is a very valid consideration, but for purposes of this paper, a company's dividend policy has been excluded from any of the analysis.

It is felt that most investors are competitive and become disillusioned if their portfolios are not performing as well as the market averages, such as the Dow Jones or the Standard and Poor's indexes. When this occurs, they will attempt to improve their performance by eliminating poor performers and buying into stocks that are performing better than average. This means that they will not be looking for other "losers" in which to invest, but rather will look for a company which they feel will improve their overall portfolio performance. At least one basic motive lies behind all investors, no matter what method or criteria they may use for investment. They are all attempting to make money or to maximize the profit on their investment, given an acceptable level of risk. They do not like to lose and will attempt to establish some measure of success. Or, as Norbert Weiner stated in God and Golem, Inc.:

> The chief criterion as to whether a line of human effort can be embodied in a game is whether there is some objectively recognizable criterion of the merit of the performance of this effort . . . [if there is no standard then] Under these circumstances, to win has no meaning, and a successful policy cannot be learned, because there is no criterion of success.

Not wishing to engage in any formal debate as to which measure or criteria is the best or most frequently cited index for comparison, this writer has chosen the Standard and Poor's 500 Composite Stock Index. This index represents approximately 90 percent of the market value of all common stocks listed on the New York Stock Exchange. It is comprised of 425 industrial common stocks, 25 rail stocks, and 50 utility stocks. In computing this index, the major companies are heavily weighted as the price of each share is multiplied by the number of shares outstanding in determining the total market value. The main reason for the choice of this index was its broad overall representation of the major industries within the American economy.

Investors often do not appear to act in a rational, logical manner when it comes to an evaluation of a company during times of either prosperity or business recession. At either end of the spectrum, the price of a company's stock can be pushed to a level that does not appear justifiable in relation to the price or value of other stocks that are in a seemingly comparable staus. Investors tend to shift from levels of optimism or pessimism, driving prices to unreasonable levels when compared to the present value or worth of the company.

## STATEMENT OF PURPOSE

It is due to the emotional over-reactionary tendency of many investors, that it was felt that a mathematical model or some other mechanistic approach for forecasting the stock market should be tested and evaluated. A tool of this sort would not be subject to personal bias and emotionalism and would only output what it was programmed to do. This is in no way advocating that personal judgment should be excluded in favor of mechanical rules. It is just stating that this approach could be used as an aid in signaling to the investor that a certain stock should be evaluated for further investment consideration.

Different mathematical or technical methods were studied and considered for a detailed evaluation of their usefulness as a forecasting tool. The three main methods considered were: multiple regression, difference equations, and exponential smoothing. All three are somewhat interrelated and could be used together, depending on the approach that one wanted to pursue.

Exponential smoothing was finally chosen as the method which would be tested for evaluation due to the following reasons:

1. This method was new and interesting to the writer, and an area in which a better understanding was desired.

2. This model has the capability to respond to change and seemed more appropriate for forecasting the stock market where radical price changes do occur.

3. The number of time periods which have an effect on the forecast could be varied by changing the smoothing constant. This method also places more emphasis on present data than it does on older data for forecasting purposes.

4. This method can rely solely on past price data and does not require the forecasting of earnings or other fundamental factors that may influence stock prices.

The main objective of this analysis, after many different approaches and ideas were tested or discarded, became one of testing and evaluating exponential smoothing with the quadratic model as a method for forecasting the price of common stocks for a variable number of time intervals.

# CHAPTER II

## DATA DESCRIPTION AND ADJUSTMENT

Before any formal tests of this model could be undertaken, it was first necessary to determine what company price data was available and for what time interval the data was to be collected. Daily, weekly, and monthly data was available, but this model tends to amplify noise that might be present within larger time intervals and it was felt that significant changes might be obscured when larger data sampling periods were used. The noise within the data will be defined as the difference between the true underlying process of price movements and the actual price data used to represent the system.

It was also felt desirable to test this model through many bull and bear markets to determine its potential use for the long-term investor, rather than just for the short-run trader or speculator. It then became a compromise between data that would cover a reasonable period of time and which would also show many of the significant price changes.

Weekly data was chosen for this analysis. The closing price for the last trading day of the week for five individual companies was collected for the time period from February 5, 1960 through June 5, 1970.

The data was collected from the financial section of the New York Times via the microfilm machine at the University of Montana library. The accuracy of the data is not guaranteed, as there were many opportunities for errors to occur, by either misprints or copying errors. An attempt was made to minimize these errors by adding the weekly net price change to the previous week's price to compare with the next week's listed closing price.

The five companies selected for analysis were: Allied Chemicals, American Motors, Anaconda Company, Boeing, and United Air Lines. These companies were chosen by a judgment sample, in that they were selling at a low price-earnings ratio and that they represented diverse industries. If the data had been more easily accessible and more adequate computer facilities had been available, more formal and reliable statistical tests would have been performed.

Information in regard to stock splits and stock dividends for each company was obtained from Moody's Financial Services. The price data was then adjusted for stock splits and dividends issued by the computer program listed in Appendix A. The adjusted price data for each of the five companies is given in Appendix B. The data has been adjusted backward in time so that all data is comparable to the most recent observation.

The exponential smoothing program is given in Appendix C. With these three appendixes, the reader may check or reproduce any part of this analysis, if desired, to verify the results. It would be necessary to plot graphically the computer's outputed results for each of the forecasting periods covered. The two programs were written in Basic FORTRAN II and were processed on the IBM 1620.

Data for the number of shares traded per week for each company was also collected for the evaluation period. This data was adjusted for stock splits and stock dividends so that the number of shares traded each week would be comparable to the total number of shares available to be traded as of the most recent observation. The mean, variance, and standard deviation of the number of shares traded per week were also calculated for each company. This variable was dropped from the analysis at this point and no further tests were made with volume figures.

Data for the Standard and Poor's 500 Composite Stock Index was also collected for the evaluation period. As it was felt that investors are competitive, these figures were also to be adjusted into a form in which tests could be conducted to determine if they would be useful to an investor. This data was tested through the exponential smoothing and forecasting section of the analysis, but at this stage it was decided to exclude this variable and no further tests were made with the index.

# CHAPTER III

## THE QUADRATIC MODEL AND EXPONENTIAL SMOOTHING*

The model chosen to represent or describe a specific system may be but an abstraction or simplification of the real world concept which it purports to duplicate. One can develop a model as complex or as simple as desired, depending on the system that is to be represented. A model should be judged mainly by its relative merits regarding its practical usefulness in representing the behavior of the actual system.

In attempting to develop a model to describe the system underlying the movement of stock prices one might want an elaborate model giving various weights to the many variables determined important in affecting the system. This approach could be followed but one should realize that an additional cost would be involved for estimating the extra coefficients of a more elaborate model. This additional cost should be balanced against the benefits derived from more accurate forecasts.

If one were inclined to believe that the system was periodic or ordered sequentially, a trigonometric model

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*Note: Reference should be made to Robert Goodell Brown's book, Smoothing, Forecasting and Prediction of Discrete Time Series, Prentice-Hall, New York; 1963, for a more formal, theoretical approach to exponential smoothing, as many of the basic concepts of this chapter are taken from his book.

composed of sine and cosine functions would be appropriate. By taking enough terms of the Fourier series, it is possible to make this model conform as closely as desired to a system of this nature. Before using a model of this type, the number of data observations per cycle or period must be known.

As neither of the two classes of models mentioned above, appeared practical or reasonable for this analysis, we will concern ourselves with the problem of approximating a given function $P = f(T)$ by means of a power series of polynomials of the general form

$$P_N(T)=a_0+a_1T+a_2T^2+ \dots +a_NT^N$$

With this function we are assuming that price is the dependent variable and is a function of the independent variable time. Let us assume that the origin of time, $T = 0$, is the present time at which we have a new data observation. We now want to determine the coefficients of the polynomial curve which best represent the price function.

More specifically, what polynomial of degree n, comes closest to fitting the given curve near T. We desire that approximating curve which passes through T and which has the highest possible degree of contact with the given curve at T. This is accomplished by expanding the function around this point T. To expand the function $P = f(T)$ around the point T, means to transform that function into a polynomial form in which the coefficients of the various terms will be expressed in terms of the appropriate derivative values, all evaluated at the point of expansion T.

It can be proven that the coefficients of the approximating polynomial which has the highest degree of contact with the given curve are:

$$a_0=P(0),\ a_1=P'(0),\ a_2=\frac{P''(0)}{2!},\ \ldots,\ a_N=\frac{P^{(N)}(0)}{N!}$$

By substituting these values into our general model, we then have for our approximating polynomial of degree n:

$$P_N(T)=P(0)+P'(0)T+\frac{P''(0)}{2!}T^2+\ \ldots\ +\frac{P^{(N)}(0)}{N!}T^N$$

This polynomial series is known as Maclaurin's series for a given function P(T), expanded around the point T = 0.

More generally, we may be concerned with expanding the polynomial function about some value of T near A, rather than around the point T = 0. It has been proven, in an analogous manner to that used for the Maclaurin's series, that the polynomial function can be expanded around any point A, not necessarily zero. This is known as the Taylor series expansion of the polynomial function P(T) about the point T = A. The general form of this equation is:

$$P_N(A)=P(A)+P^{(1)}(A)(T-A)+\frac{P^{(2)}(A)}{2!}(T-A)^2+\ \ldots\ +\frac{P^{(N)}(A)}{N!}(T-A)^N+\ \ldots$$

where $P^{(N)}(A)$ denotes the value of the Nth derivative of the function P(T) at T = A. The equation given above requires that the function possess finite derivatives of all orders at T = A.

The Maclaurin and Taylor polynomial power series have shown that a function can be represented by an expansion about any point desired, so long as the coefficients of the model can be approximated by the appropriate order derivatives. The results of these series were developed for continuous functions using differential calculus; but these results can also be applied to discrete time series. A detailed theoretical discussion on the analogies between Difference Equations and Differential Calculus will not be given. Readers interested in a comparative analysis of the operators of these two mathematical disciplines may reference a book in this area, such as Samual Goldberg's book, Introduction to Difference Equations.

At this stage of the analysis, it was necessary to determine the order of the polynomial model that would be useful in describing and forecasting the underlying system of price changes. For any given period of time, the price of a stock might be very stable and a constant model would be adequate for forecasting purposes. At other times, the price might be changing at a constant rate and a linear model would be appropriate. If the price was accelerating at a constant rate, then the quadratic model would be more appropriate. A cubic function of time would be pertinent if the acceleration was changing linearly.

Theoretically, if the data can be represented by a simple average over time, then the first differences between successive observations should fluctuate randomly around zero. If a straight line would be a better model, then the first differences will have a non-zero average, but the second differences will fluctuate around zero. In general, if the process underlying the observations is exactly represented by a polynomial of degree n, then it is possible to find that degree by successive differences. The $n + 1$ difference will be identically zero, if the polynomial is of degree n. Usually one never has data of the underlying process itself, but only noisy observations and the ($n + 1$)st differences will average zero.

The problem of determining the correct model was even further complicated because the coefficients in the underlying process are not literally constant, but may undergo a slow random walk in time. It is assumed that this change is small enough so that, at any one point in time, the current estimates of the coefficients will provide a satisfactory forecast.

The quadratic model was chosen for this analysis. It was felt that this polynomial model, in regards to accuracy; was the most practical and adequate one to use in representing the underlying process of price movements. The choice of this model confirms the theoretical results, as it is generally accepted that the third differences of stock prices have a zero average.

This model is of the second order and includes the first three terms of the Taylor series. Forecasts are made T periods ahead with this model by the following equation:

$$S(T)=A+BT+1/2\ CT^2$$

where S(T) is the forecast of the future price T periods ahead of the present time on which data is available to determine the coefficients A, B, and C of the model. One of the principal objectives of an adaptive system is to revise the coefficients of the model after each new data observation. Then new forecasts are made based on the revised coefficients.

Exponential Smoothing is the process which has been chosen for revising the coefficients of the quadratic model after each new data observation. This method is appropriate for forecasting the price of stocks as the data is from a discrete time series. The prices are in distinct, specific levels and the data may be gathered or obtained at the end of a specified time interval.

Exponential smoothing, although not exactly the same, is very comparable to a weighted moving average. The smoothing function is a linear combination of all past observations with the smoothing constant determining the weight given to each observation. The heaviest weight is given to the most recent observation and the weight given to previous observations decreases exponentially with age.

When the smoothing constant is small, the function behaves similarly to a moving average that includes a large number of observations. The larger the smoothing constant, the fewer are the number of observations that have a significant effect on the forecast and the function will respond very rapidly to a change in the process. When a smoothing constant is to be selected, one is basically faced with determining whether the process is relatively stable and a lower value will suffice, or whether the process is changing and a higher value of the constant is required.

In this paper, exponential smoothing uses a least-squares criterion of accuracy. The values of the coeffecients are computed in such a way as to minimize the sum of the squares of the difference between the actual observation and the value that was forecasted by the model. These differences are also referred to as residuals.

Exponential smoothing is the operation performed on any sequence of observations in order to obtain recursive revisions to the previous estimates of the coefficients of the model. The smoothing function used in this analysis requires initial values for the coefficients of the quadratic model. The first forecast is made based on these initial coefficients. Once the next data observation is obtained the smoothing process to revise the coefficients can begin.

Single exponential smoothing is the method used to estimate the value of the constant term of the model. The new smoothed value is equal to the previous smoothed value plus a fraction (the smoothing constant) of the difference between the new observation and the previous smoothed value.

Double exponential smoothing is the process used to obtain the new estimate of the linear term of the model. Double smoothing is the result of applying the smoothing process to the single smoothed results.

Triple exponential smoothing is just the smoothing process applied to the double smoothed results, and is used to estimate the quadratic term of the model.

With these three new estimates of the coefficients, the next forecast is made. The smoothing process is repeated as each new observation is received. This is the exponential smoothing sequence that was used for this analysis and the computer program located in Appendix C.

This model, like many other mathematical models, follows certain basic assumptions regarding its form. It is generally assumed that the actual data observations contain noise and are not exactly equal to the underlying process. It is also assumed that the residuals or forecast errors are normally distributed with a mean of zero and a constant variance and that the actual data samples are not serially correlated.

# CHAPTER IV

## ANALYSIS OF THE MODEL

In order to test this model, some objective measure of success or failure had to be established. It is probably generally accepted that one would desire a model that accurately forecasted the future price of a stock.

When using an exponential smoothing function, it was believed that if too many observations were included, due to a low smoothing constant, the forecast error would be large as the model would not respond to a change in the system. Conversely, if the smoothing constant was too large, the model would respond to minor changes, causing the error to increase. Due to this belief, it was decided to vary the smoothing constant to determine the effect that this would have on the forecast errors.

The mean of the residuals is one criteria of accuracy that might be employed. As this measure is determined by summing all the forecast errors, large positive errors would be offset by large negative errors. This would cause the mean of the forecast errors to be a rather meaningless measure for comparison.

In order to overcome this limitation, the minimum <u>variance</u> estimator of the smoothing constant was used as the measure of accuracy for this paper. Another quality of this measure is that it gives more weight to large errors which one would like to eliminate.

The original exponential smoothing program used was a scientific subroutine written by IBM. This program was modified so that the smoothing constant could be varied through any range from 0 to 1.0 by any three-digit decimal change that was desired by the user.

The first test that was conducted was a one week forecast for each of the five companies. An initial value for the constant term of the quadratic model was read into the computer along with the actual data to begin the smoothing and forecasting process. The computer program was also modified so that the first 100 observations were used to permit the model to settle and to adjust the coefficients so that they would be a better representation of the actual process. 440 observations, for the time period from January 5, 1962 through June 5, 1970, were used in the analysis to determine the minimum variance estimator of the smoothing constant.

The smoothing constant was initially varied by an increment of .05 through the range from 0.0 to 0.50 for one company. The model reacted as expected with the error large for a very small constant, then decreasing as the constant was increased until it reached a smoothing constant of .300. At this point the error increased as the smoothing constant was increased.

It was then decided to test all five companies for this one week forecast by varying the constant by an increment of .025 through the range from .275 through .350. In all cases, the model, reacted as expected and a smoothing constant of either .300 or .325 produced the minimum variance estimate. In this range there did not appear to be a significant difference in the results for either of these two values. The forecasts and residuals based on a smoothing constant of .325 were selected for future analysis.

In order to analyze the output of the model for this smoothing constant and forecast period, it was next necessary to graph the results. The number of shares traded each week and the standard deviation of the trading activity were also entered on each graph. At this point of the analysis it did not appear that the one week forecast alone would be a useful aid to an investor. With a smoothing constant of .325 for triple smoothing, only two data observations have a significant effect in determining the next forecast.

The next conclusion reached was that this same analysis could be extended by forecasting for periods further into the future. It was also assumed that the further one forecasted ahead, the larger the minimum variance estimator would become. Later tests have verified the validity of these assumptions.

Before this approach could be tested, the computer program had to be modified to (1) include the capability of forecasting ahead for a variable number of time periods, and (2) to include exponent notation for every variable that might increase to a quantity too large to be output on the existing computer available. This is the final Exponential Smoothing program that is given in Appendix C at the end of this paper.

Initially, one company was tested by varying the smoothing constant through the range from .050 through .300 by increments of .050 for each forecast period 2 through 6 inclusive. From this initial test, a separate range for the smoothing constant was established for each forecast period, so that further tests could be conducted on all five companies. This procedure does reduce the operating time and narrows down the number of smoothing constants to be tested.

During the initial test when forecasting for six weeks ahead, the forecast error for every smoothing constant tested, appeared to become too large to be of any value to an investor so this forecast period was disregarded from any further analysis.

The summary results for each forecast period (T) for the five companies tested and the smoothing constant (AL) for which the deviation of the forecast errors was a minimum are given in Table 1. A detailed analysis of these results will not be given, but a few points of significance will be mentioned.

This model was based on the assumption that the forecast errors were normally distributed with a mean of zero and a constant variance. For the forecasts up to three periods ahead, these assumptions do not appear to be violated; but when forecasting four and five periods ahead, the forecasts do not appear to meet the established criteria.

According to expectation, the deviation of the forecast errors increased the further one forecasted into the future. This increase was not linear in relation to the length of the forecast period, but rather consistently the largest increase in the deviation of the forecast errors occurred between three and four forecast periods ahead.

Before concluding this section, two final points of interest will be mentioned about this model. First, when forecasts were made further than one week ahead, the smoothing constant appeared to become more sensitive. For a smoothing constant of .200 or greater and the forecast period 2 or larger, the minimum variance estimator was greater than ten to the forty-fifth power, making the results useless to an investor. Second, when forecasting four or five periods ahead with weekly data, the deviation of the forecast errors tends to become sensitive to small changes of the smoothing constant.

TABLE 1

SUMMARY SHEET FOR PRICE FORECASTING AND FORECAST ERROR

| | SUM TOTAL OF FORECAST ERRORS | TOTAL OF SQUARED FORECAST ERRORS | MEAN OF FORECAST ERRORS | VARIANCE OF FORECAST ERRORS | ST. DEV. OF FORECAST ERRORS |
|---|---|---|---|---|---|
| ALLIED CHEMICALS | | | | | |
| T=1; AL=.325 | 1.977 | 819.756 | .004 | 1.867 | 1.366 |
| T=2; AL=.125 | -3.286 | 3022.628 | -.007 | 6.885 | 2.623 |
| T=3; AL=.100 | -25.328 | 7292.255 | -.058 | 16.611 | 4.075 |
| T=4; AL=.040 | -83.760 | 38721.441 | -.190 | 88.204 | 9.392 |
| T=5; AL=.040 | -212.726 | 64702.839 | -.483 | 147.387 | 12.140 |
| AMERICAN MOTORS | | | | | |
| T=1; AL=.325 | -.065 | 287.661 | -.000 | .655 | .809 |
| T=2; AL=.125 | .499 | 1300.235 | -.001 | 2.961 | 1.720 |
| T=3; AL=.100 | -3.687 | 2366.995 | -.008 | 5.391 | 2.322 |
| T=4; AL=.030 | 27.248 | 2546.955 | -.062 | 5.802 | 2.409 |
| T=5; AL=.030 | 6.667 | 5056.966 | -.015 | 11.519 | 3.394 |
| ANACONDA | | | | | |
| T=1; AL=.325 | -.618 | 1315.291 | -.001 | 2.996 | 1.731 |
| T=2; AL=.125 | 4.558 | 5579.502 | .010 | 12.710 | 3.565 |
| T=3; AL=.100 | 6.044 | 15554.728 | .013 | 35.432 | 5.952 |
| T=4; AL=.040 | -280.293 | 56473.955 | .637 | 128.642 | 11.342 |
| T=5; AL=.040 | -301.391 | 103678.460 | -.685 | 236.170 | 15.368 |
| BOEING | | | | | |
| T=1; AL=.325 | 3.196 | 3945.925 | .007 | 8.988 | 2.998 |
| T=2; AL=.125 | -9.154 | 11026.905 | -.020 | 25.118 | 5.011 |
| T=3; AL=.100 | -40.665 | 16679.094 | -.092 | 37.993 | 6.163 |
| T=4; AL=.030 | -89.505 | 117375.420 | -.203 | 267.370 | 16.351 |
| T=5; AL=.030 | -195.233 | 152498.020 | -.444 | 347.376 | 18.638 |
| UNITED AIR LINES | | | | | |
| T=1; AL=.325 | 4.198 | 2946.351 | .010 | 6.712 | 2.591 |
| T=2; AL=.125 | 7.838 | 9507.771 | .017 | 21.657 | 4.653 |
| T=3; AL=.100 | 2.808 | 21777.812 | .006 | 49.607 | 7.043 |
| T=4; AL=.030 | 266.402 | 70686.647 | .605 | 161.017 | 12.689 |
| T=5; AL=.030 | -59.356 | 164870.460 | -.135 | 375.559 | 19.379 |

NOTE: T = forecast period

AL = smoothing constant

# CHAPTER V

## TEST FOR PROFITABILITY

In any analysis of this type, it would appear to be incomplete without some test of the profitability in using the results derived from the model for investment purposes. With the forecasts for five different time periods, it became not only a problem of establishing certain criteria for each forecast period that was selected to be used in the analysis, but also in determining the specific type of investor for which its use would be beneficial as a tool for investment decision-making.

If one were inclined to be a short-run trader, trying to catch the minor swings in the market, the one week forecast would be more appropriate.

One method that could be used for this type of investment is to treat the forecast as a tracking signal, in that every time the forecasted price is less than the actual price, a buy signal is indicated. Once the stock is bought, it is only sold once the forecasted price becomes larger than the actual price. With a smoothing constant of .325, the model is very responsive and one would be in and out of the market excessively. Tests could be run to determine the smoothing constant that would maximize profits as the main objective, rather than to minimize the forecast error. A recommendation might be to test the model by varying the smoothing constant by increments of .025 through the range from .150 to .325 for weekly data. By using this test on one company's data, an acceptable range could be established before testing the other four companies. This would reduce the amount of operational time required and could serve as a future guide for companies of similar homogeneous price movements.

The two or three week forecasts could be tested in this same manner and would be useful to the intermediate trader who would not be interested in changing his portfolio quite so frequently.

The following factors were considered in selecting the test for profitability. First, with the previous tests that had already been completed and the data plotted graphically, it was felt that some useful criteria could be established that would aid the long term investor in selecting stocks for investment. Second, is the fact that the federal income tax is more favorable to the long term investor; third, broker's fees can become a very significant cost to the common stock investor.

After analyzing the data, it became apparent that it would be beneficial to use a combination of the forecasts in establishing some fixed investment rules, rather than using one individual forecast, such as the four or five week forecast. Any of the five, used individually, did not appear to be profitable.

At this stage of the analysis, it was felt that it would be a rather simple task to establish some investment rules, but such was not the case. It was finally decided that separate rules should be established for buying, holding, and selling stock.

These investment rules were determined in the following manner. For each company, major price movements were identified and classified into one of the following categories: increasing, decreasing, constant or fluctuating. From these major categories of price movements, areas of favorable and unfavorable investment potential were identified. Then, for each forecast period and the actual price data, major identifying trend characteristics were tabulated for these two categories of investment potential. From this list of trend characteristics, initial investment criteria were established for the actual data and the forecast periods to be used in the test for profitability. Initial tests were made and modifications were made to the criteria.

The final investment criteria used are the results of many tests and modifications that were made in an attempt to eliminate unfavorable transactions. They were purposefully made stringent, in an attempt to reduce the risk of loss. These stringent criteria have eliminated many favorable, as well as unfavorable transactions. No one is advocating that the following criteria that were established are the best or the most profitable ones that could be used.

Criteria for Buying, Holding, or Selling Stock

The five week forecast was the one chosen to be the main criteria for the determinant of investment decision making. Both the four and five week forecasts tended to move together and penetrate or cross the actual price line within a few time intervals. At times, when the price was either increasing or decreasing rapidly, but fluctuating in an increasing and then decreasing manner, these two forecasts tended to drift apart and become erratic. Anytime this occurred, it did not appear feasible for a long term investor to buy or enter into the market. A one or two week forecast would probably be more appropriate at this time.

This erratic behavior of the model at longer forecasting periods is probably due to the sensitivity of the model exhibited in a large change in the total forecast error for a small change in the smoothing constant. This characteristic could be checked at a computer facility that had plotting capability, by varying the smoothing constant by an interval of .005 through the range of .025 to .060; and then analyzing the plotted output.

In any case, these forecasts tended to cross the actual price line at points where there was a change in the price movement of the stock. In other words, the price system was changing from a declining movement to either a constant process or an increasing movement. It is this characteristic of the five week forecast that will be used in establishing investment criteria.

Anytime the sign of the five week forecast error changes, from either positive to negative or vice versa, a buy, hold, or sell rule is established. In order that random changes during an erratic price movement do not signal an investment decision, the following rule is set for the five week forecast: The forecast must be increasing or decreasing for five consecutive periods, the fifth period to include the period in which the change in the sign of the forecast error occurs. In addition to the above established rule, investment decisions are subject to the following constraints:

Buy Criteria

1. Actual Price Data - The price of the stock in the period in which the change in the sign of forecast errors occurred, must be greater than the actual price was six periods previous to the change.

2. Three Week Forecast - This forecasted price must be less than the corresponding actual price was in at least four of five consecutive periods; the last period is to be the period in which the change occurred.

3. Four Week Forecast - There must be one and only one change in the sign of the forecast errors for this forecasting interval within six time periods from which the change in the five week forecast occurred.

4. Five Week Forecast - The sign of the forecast errors must not have changed within the previous ten time periods from the period under consideration.

Hold Criteria

1. Actual Price Data - This price in the period in which the sign of the forecast errors changes must be greater than the actual price was six periods previous.

2. Three Week Forecast - This forecast in the period in which the change occurred must be either less than the actual price or greater than the previous three week forecast.

3. In order to insure stability of the system for a hold decision, it is further assumed that BUY Criteria No. 3 must be satisfied. If this criteria is not satisfied, then the stock is to be sold at the end of the sixth period following the change in sign of the errors for the five-week forecast.

Sell Criteria

EITHER 1. Actual Price Data - This price in the period in which the sign of the forecast errors changes must be less than the actual price was six periods previous.

OR 2. Three Week Forecast - This forecast in the period in which the change occurred must be both greater than the actual price and less than the three week forecast was three periods prior to the change.

In accordance with the above established investment criteria it is assumed that for each transaction that takes place, the following assumptions are met:

1. One hundred shares of stock are traded at each transaction, with a maximum of 100 shares of any one company to be held at one time. It is assumed that when a buy decision is undertaken, the investor allots the total amount he is willing to invest in that company at one time.

2. A one percent broker's transaction fee will be charged for each buy decision.

3. A simple annual rate of return on investment will be calculated for each transaction, based on the difference between the total value of the stock at selling time and at buying time, less broker's fees. This difference is first divided by the purchase price of the stock, and then by the number of years the stock was held to yield the annual return on investment.

## Analysis of Financial Transactions

Each of the five companies were checked individually to determine the number of transactions, if any, that would satisfy all of the above-mentioned financial criteria.

A total of six transactions occured for these five companies, but one is still pending on the future before its profitability can be determined. American Motors is the sole company for which no transactions were recorded. This is due to the fact that it did not have any substantial changes in its price over any period of time. All of the transactions are summarized in Table 2.

TABLE 2

SUMMARY OF FINANCIAL TRANSACTIONS

| COMPANY TRANSACTIONS | BUY PERIOD | BUY PRICE | HOLD PERIOD | SELL PERIOD | SELL PRICE | WEEKS STOCK HELD | DIFFERENCE BUY AND SELL | DIFFERENCE LESS BROKER'S FEES | ANNUAL RETURN ON INVEST-MENT |
|---|---|---|---|---|---|---|---|---|---|
| AMERICAN MOTORS | | | | | | | | | |
| 1. NO TRANSACTIONS | | | | | | | | | |
| ALLIED CHEMICALS | | | | | | | | | |
| 1. | 175 | $46\frac{7}{8}$ | -- | 219 | $53\frac{3}{4}$ | 44 | $6\frac{7}{8}$ | $6\frac{3}{8}$ | 16.1% |
| ANACONDA | | | | | | | | | |
| 1. | 267 | $32\frac{1}{2}$ | 310 | 316 | 46 | 49 | $13\frac{1}{2}$ | $13\frac{1}{8}$ | 42.8% |
| 2. | 511 | 31 | ? | ? | ? | ? | ? | ? | |
| BOEING | | | | | | | | | |
| 1. | 214 | $21\frac{3}{8}$ | 287 | 327 | $71\frac{7}{8}$ | 113 | $50\frac{4}{8}$ | $50\frac{1}{4}$ | 108.1% |
| UNITED AIR LINES | | | | | | | | | |
| 1. | 133 | $14\frac{5}{8}$ | --- | 188 | $20\frac{1}{2}$ | 55 | $5\frac{7}{8}$ | $5\frac{3}{4}$ | 34.8% |
| 2. | 256 | $30\frac{1}{4}$ | 316 | 322 | $69\frac{1}{8}$ | 66 | $38\frac{7}{8}$ | $38\frac{4}{8}$ | 100.1% |
| TOTALS | | $145\frac{5}{8}$ | | | $261\frac{1}{4}$ | 327 | $115\frac{5}{8}$ | 114 | |
| AVERAGES | | $29\frac{1}{8}$ | | | $52\frac{1}{4}$ | 65.4 | $23\frac{1}{8}$ | $22\frac{1}{8}$ | 62.1% |

The annual return on investment for the individual transactions ranged from 16.1 percent to 108.1 percent. The average annual return for the five transactions was 62.1 percent. This figure is based on the assumption that the investor remains fully invested for the entire time period that stock is held. This return may be of some significance, but its reliability in reference to any other companies than the five tested is a point of conjecture. If the data was available, a random sample could be established and these companies tested against these criteria to determine the profitability for any transactions that occurred.

It was felt that some comparative measure should be established against these transactions for the total time period that stock was held. The first stock was purchased at the end of period 133, and the last stock was sold at the end of period 327.

The Standard and Poor's 500 Composite Stock Index was chosen as the mode with which a comparative measure would be established. This index stood at 58.66 at the end of period 133, and increased to 89.39 at the end of period 327.

The annual return on investment for this index was 14.0 percent as compared to 62.1 percent for the average annual return per transaction. These percentages are not directly comparable as some of the transactions were overlapping with more than one company's stock held at one time, but it is the best comparative measure that was developed for this paper.

# CHAPTER VI

## SUMMARY

The results of this analysis are not conclusive. All of the possible approaches or avenues of use for this model have not been tested nor exhausted. This analysis has been one approach that an investor might use as a tool to aid in selecting or changing his portfolio of common stocks.

The practical and theoretical appropriateness of the quadratic polynomial model as the mode for forecasting have been discussed. Also, the reasoning for the choice of exponential smoothing as the means of revising the coefficients of an adaptive model has been given.

The criteria of accuracy used in this analysis was to determine the smoothing constant for each forecast period that was the minimum variance estimator. This is a good relative measure of forecast accuracy, but one may also desire to establish another criteria of investment success. It is assumed that most investors would prefer to maximize the return on their investment, not just make a profit. This could become the primary objective of a future study.

The investment criteria that were established for this analysis are for the conservative investor who has an aversion to risk. Many more investment opportunities would be available if these criteria were relaxed. This writer would not personally use this set of investment rules; but with the use of adequate computer facilities, would conduct more formal tests to determine a combination of forecast periods that would optimize profits.

One approach toward solving this problem would be to vary the smoothing constant, using each forecast period individually, to determine the constant that would optimize profits. Initially, the investment criteria could be simple. The forecast could be used as a tracking device, signaling the investor to buy when the forecast was less than the actual price, and sell when the forecast was greater than the actual price. Further tests could include the use of combinations of forecast periods or alterations to the investment criteria. As there are a multitude of approaches that could be used at this stage, they will be left to the readers discretion.

One would not have to use weekly data exclusively, but could also test the use of combinations of daily, weekly and monthly data for the various forecast periods. The exponential smoothing program given could be modified or changed at the user's discretion in order to test any approach deemed appropriate.

As only five companies were tested, the results obtained from this analysis may not be comparable to those that would be obtained from a similar study for a larger sample of companies. No alterations or modifications of the data have been made with the intent of making the results look impressive.

A final personal opinion in regards to the investment criteria established for this analysis is as follows: the results obtained from a larger sample using weekly data would be very comparable to those found in this analysis.

## BIBLIOGRAPHY

Bellemore, Douglas H. and Ritchie, John C. Jr. Investments Principles, Practices, Analysis. South-Western Publishing Company, Cincinnati, Ohio, 1969.

Brown, Robert Goodell. Smoothing, Forecasting, and Prediction of Discrete Time Series. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1963.

Carpenter, H.G. Investment Timing By Formula Plans. Harper and Brothers Publishers, New York, 1958.

Chiang, Alpha C. Fundamental Methods of Mathematical Economics. McGraw-Hill Book Company, New York, 1967.

Draper, N.R. and Smith, H. Applied Regression Analysis. John Wiley and Sons, Inc., New York, 1966.

Edwards, Robert D. and Magee, John Jr. Technical Analysis of Stock Trends. Stock Trend Service, Springfield, Mass., 2nd edition, 1951.

Farrar, Donald Eugene. The Investment Decision Under Uncertainty. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1962.

Fisher, Philip A. Common Stocks and Uncommon Profits. Harper and Brothers Publishers, New York, 1958.

Goldberg, Samuel. Introduction to Difference Equations. John Wiley and Sons, Inc., New York, 1958, Fourth Printing, May, 1967.

Kimmel, Lewis H. Share Ownership in the United States. The Brookings Institution, Washington 6, D.C., 1952.

Kuh, Edwin. Capital Stock Growth: A Micro-Econometric Approach. North-Holland Publishing Company, Amsterdam, 1963.

McCameron, Fritz A. Fortran Logic and Programming. Richard D. Irwin, Inc., Homewood, Illinois, 1968.

Meiselman, David. The Term Structure of Interest Rates. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1962.

Smith, Adam. The Money Game. Dell Publishing Co., Inc., New York, N.Y., 1969.

Thomas, George B. Jr. Calculus and Analytic Geometry. Addison-Wesley Publishing Company, Inc., Reading, Mass., 1953.

Wiener, Norbert. God and Golem, Inc. The M.I.T. Press, Massachusetts Institute of Technology, Cambridge, Mass., 1964.

Wonnacott, Thomas H. and Ronald J. Introductory Statistics. John Wiley and Sons, Inc., New York, 1969.

# APPENDIX A

```
C     PROGRAM FOR DATA ADJUSTMENT FOR STOCK SPLITS AND STOCK DIVIDENDS
C
C     FIRST CARD SHOULD INCLUDE NSPLIT, NDIV, AND NX. THEN DATA DECK
C     MUST READ IN PARAMETER CARDS STATING MAGNITUDE OF SPLITS
C     AND DIVIDENDS AND TIME PERIOD IN WHICH EACH OCCURRED.
C
C
C     TABLE OF VARIABLES
C     NSPLIT = NUMBER OF STOCK SPLITS
C     VSPLIT = VALUE OF STOCK SPLIT
C     NS = PERIOD IN WHICH STOCK SPLIT OCCURRED
C     NDIV = NUMBER OF STOCK DIVIDENDS
C     VDIV = VALUE OF STOCK DIVIDEND
C     ND = PERIOD IN WHICH STOCK DIVIDEND WAS ISSUED
C     NX = NUMBER OF PERIODS OF ACTUAL DATA
C      X = ORIGINAL DATA
C
C     PROGRAM
C
      DIMENSION X(600),NS(100),ND(100),VSPLIT(100),VDIV(100)
  100 FORMAT(I2,I2,I4)
  101 FORMAT(10(F7.1,1X))
  102 FORMAT(F2.1,I4)
  103 FORMAT(F3.2,I4)
  104 FORMAT(40H THIS IS END OF THIS COMPANY'S DATA DECK)
C
C
    1 READ 100,NSPLIT,NDIV,NX
      READ 101,(X(N),N=1,NX)
      IF(NSPLIT)35,35,2
    2 DO 3 I=1,NSPLIT
      READ 102,VSPLIT(I),NS(I)
    3 CONTINUE
   35 IF(NDIV)6,6,4
    4 DO 5 I=1,NDIV
      READ 103,VDIV(I),ND(I)
    5 CONTINUE
    6 IF(NSPLIT)7,9,7
    7 DO 8 I=1,NSPLIT
      NNS = NS(I)
      VVSPL = VSPLIT(I)
      DO 8 K=1,NNS
      X(K) = X(K)/(VVSPL)
    8 CONTINUE
    9 IF(NDIV)10,12,10
   10 DO 11 J =1,NDIV
      NND = ND(J)
      VVDIV = VDIV(J)
      DO 11 L=1,NND
      X(L) = X(L)/(1.0 + VVDIV)
   11 CONTINUE
   12 PUNCH 101,(X(N),N=1,NX)
   13 PUNCH 104
      GO TO 1
      END
```

APPENDIX B

ALLIED CHEMICALS- ADJUSTED PRICE DATA

TIME PERIOD COVERED- FROM FEB 5,1960 TO JUNE 5,1970
WEEKLY DATA LISTED BY 10 PERIODS PER ROW
READ BY ROW- STARTING AT TOP LEFT FOR FEB 5,1960

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 48.2 | 46.9 | 47.2 | 50.4 | 47.6 | 49.2 | 46.7 | 48.5 | 46.2 | 47.1 |
| 47.1 | 46.2 | 46.6 | 46.7 | 47.1 | 48.1 | 48.8 | 49.6 | 51.6 | 51.9 |
| 53.2 | 53.2 | 51.1 | 50.7 | 50.9 | 50.9 | 50.1 | 50.5 | 51.4 | 53.7 |
| 53.0 | 51.9 | 50.5 | 47.1 | 45.1 | 47.1 | 46.6 | 46.2 | 46.2 | 48.2 |
| 49.0 | 49.4 | 49.0 | 46.9 | 49.8 | 51.8 | 50.9 | 51.3 | 48.5 | 51.0 |
| 53.2 | 53.0 | 55.8 | 55.8 | 55.6 | 53.0 | 53.7 | 54.7 | 55.2 | 56.1 |
| 58.9 | 58.0 | 58.0 | 57.5 | 55.2 | 55.1 | 55.1 | 57.9 | 58.4 | 60.2 |
| 59.9 | 59.0 | 57.9 | 55.4 | 57.1 | 58.0 | 59.6 | 60.3 | 61.8 | 60.3 |
| 60.8 | 58.6 | 58.2 | 58.6 | 56.5 | 56.0 | 55.4 | 58.3 | 54.7 | 55.1 |
| 53.7 | 52.8 | 53.7 | 52.8 | 49.9 | 51.1 | 49.9 | 52.0 | 51.4 | 52.4 |
| 53.1 | 51.7 | 50.1 | 48.5 | 50.1 | 51.0 | 49.4 | 47.6 | 48.2 | 47.5 |
| 47.0 | 45.6 | 44.1 | 44.1 | 43.7 | 43.3 | 41.0 | 42.0 | 40.0 | 38.8 |
| 37.8 | 38.8 | 39.4 | 36.0 | 34.1 | 36.7 | 36.0 | 36.7 | 34.1 | 34.4 |
| 34.7 | 34.4 | 35.2 | 35.8 | 35.9 | 35.4 | 36.8 | 35.6 | 35.1 | 35.1 |
| 35.7 | 34.5 | 33.8 | 36.8 | 38.8 | 39.0 | 40.7 | 40.6 | 39.8 | 39.8 |
| 40.7 | 41.5 | 41.3 | 42.8 | 41.1 | 41.1 | 42.6 | 42.2 | 42.3 | 41.9 |
| 40.0 | 42.2 | 42.2 | 41.9 | 42.5 | 45.6 | 45.1 | 45.2 | 46.2 | 48.2 |
| 49.6 | 46.9 | 46.4 | 46.9 | 46.9 | 48.2 | 47.6 | 45.7 | 46.4 | 46.6 |
| 45.3 | 45.0 | 47.3 | 47.6 | 47.1 | 47.7 | 47.1 | 48.2 | 49.4 | 49.1 |
| 48.0 | 47.9 | 48.4 | 50.0 | 50.9 | 50.8 | 50.8 | 50.3 | 46.2 | 51.0 |
| 51.9 | 53.0 | 53.0 | 51.8 | 52.8 | 53.5 | 52.0 | 53.1 | 50.9 | 51.8 |
| 50.9 | 50.7 | 50.8 | 51.4 | 51.8 | 53.7 | 52.9 | 53.6 | 53.7 | 52.0 |
| 50.4 | 50.9 | 51.5 | 50.3 | 49.1 | 50.4 | 49.5 | 50.0 | 49.9 | 50.5 |
| 51.7 | 51.1 | 51.6 | 51.0 | 49.8 | 49.0 | 48.8 | 49.8 | 49.4 | 50.0 |
| 49.1 | 49.2 | 50.4 | 50.5 | 50.8 | 48.8 | 49.0 | 51.9 | 52.0 | 51.4 |
| 51.0 | 49.2 | 50.1 | 49.5 | 49.4 | 48.8 | 48.6 | 50.5 | 51.3 | 53.0 |
| 52.9 | 54.8 | 52.4 | 54.6 | 54.6 | 54.9 | 54.2 | 53.1 | 53.4 | 53.6 |
| 54.2 | 54.6 | 54.3 | 51.8 | 52.3 | 52.4 | 51.7 | 49.8 | 48.1 | 46.9 |
| 46.3 | 46.0 | 46.7 | 45.8 | 45.7 | 44.8 | 44.8 | 45.4 | 45.4 | 45.0 |
| 45.9 | 48.2 | 47.7 | 49.9 | 48.0 | 47.0 | 47.5 | 46.7 | 45.7 | 45.8 |
| 45.8 | 44.6 | 45.2 | 44.7 | 46.2 | 45.6 | 46.1 | 47.7 | 48.1 | 50.8 |
| 48.5 | 47.9 | 47.8 | 46.9 | 46.7 | 45.9 | 44.9 | 45.6 | 44.9 | 44.0 |
| 43.6 | 43.3 | 44.2 | 43.2 | 42.7 | 42.3 | 40.6 | 40.2 | 39.7 | 39.2 |
| 38.6 | 39.4 | 39.0 | 39.1 | 38.3 | 39.0 | 38.2 | 37.5 | 37.5 | 36.9 |
| 36.2 | 35.4 | 34.4 | 36.2 | 35.1 | 35.7 | 34.8 | 34.2 | 32.5 | 33.4 |
| 33.3 | 34.4 | 33.8 | 33.4 | 33.7 | 32.5 | 31.9 | 33.8 | 33.3 | 33.4 |
| 33.0 | 35.9 | 37.5 | 38.2 | 41.0 | 41.1 | 39.1 | 40.2 | 40.5 | 39.6 |
| 38.9 | 40.1 | 40.9 | 40.2 | 40.0 | 40.0 | 40.1 | 42.1 | 41.8 | 41.2 |
| 39.8 | 39.2 | 39.4 | 38.6 | 38.8 | 38.4 | 37.8 | 38.4 | 39.4 | 39.1 |
| 39.2 | 40.0 | 41.8 | 43.4 | 41.5 | 42.5 | 44.6 | 45.1 | 44.2 | 43.9 |
| 43.4 | 42.6 | 41.0 | 39.8 | 39.0 | 38.1 | 38.9 | 39.2 | 38.5 | 39.0 |
| 38.5 | 39.1 | 40.0 | 41.8 | 42.1 | 40.4 | 39.4 | 38.2 | 37.2 | 36.5 |
| 36.8 | 36.0 | 35.1 | 34.5 | 34.9 | 34.9 | 34.6 | 35.2 | 36.8 | 36.2 |
| 38.4 | 36.9 | 35.6 | 35.8 | 36.0 | 36.5 | 36.9 | 35.4 | 35.2 | 35.9 |
| 37.0 | 35.1 | 35.4 | 34.9 | 34.9 | 35.5 | 35.4 | 35.0 | 37.5 | 36.0 |
| 36.1 | 36.4 | 36.0 | 35.9 | 35.4 | 37.8 | 33.8 | 34.2 | 34.0 | 36.5 |
| 36.1 | 38.0 | 36.2 | 35.0 | 35.8 | 37.4 | 36.0 | 35.9 | 36.0 | 35.2 |
| 34.8 | 35.0 | 33.2 | 31.6 | 31.9 | 32.0 | 31.1 | 30.8 | 29.8 | 30.4 |
| 31.4 | 30.2 | 33.2 | 36.0 | 34.8 | 33.0 | 33.5 | 32.8 | 30.5 | 29.5 |
| 29.0 | 29.0 | 28.9 | 28.6 | 27.2 | 29.0 | 27.6 | 27.1 | 27.0 | 26.9 |
| 26.1 | 26.0 | 26.5 | 26.0 | 26.0 | 28.0 | 30.9 | 30.0 | 30.1 | 30.2 |
| 29.4 | 28.5 | 28.1 | 26.8 | 24.4 | 24.5 | 24.8 | 25.1 | 26.2 | 25.2 |
| 25.9 | 23.5 | 23.0 | 23.0 | 22.9 | 23.5 | 24.1 | 24.1 | 23.5 | 24.2 |
| 24.1 | 23.2 | 21.0 | 20.6 | 20.0 | 19.0 | 18.2 | 17.4 | 19.8 | 18.4 |

AMERICAN MOTORS- ADJUSTED PRICE DATA

TIME PERIOD COVERED- FROM FEB 5,1960 TO JUNE 5,1970
WEEKLY DATA LISTED BY 10 PERIODS PER ROW
READ BY ROW- STARTING AT TOP LEFT FOR FEB 5,1960

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 27.0 | 25.7 | 24.0 | 23.5 | 25.3 | 23.0 | 22.5 | 24.7 | 24.9 | 26.6 |
| 28.3 | 27.3 | 25.9 | 26.1 | 25.9 | 24.3 | 23.3 | 23.7 | 23.7 | 22.6 |
| 21.6 | 22.6 | 21.8 | 21.0 | 21.7 | 22.2 | 21.0 | 21.2 | 23.0 | 22.5 |
| 22.7 | 22.0 | 21.4 | 20.6 | 21.0 | 20.6 | 20.7 | 19.8 | 19.5 | 19.6 |
| 20.0 | 19.2 | 20.0 | 19.1 | 18.6 | 17.5 | 17.7 | 17.2 | 17.6 | 17.8 |
| 17.8 | 17.6 | 16.5 | 16.8 | 17.5 | 17.6 | 18.4 | 17.3 | 19.6 | 19.7 |
| 20.5 | 20.6 | 19.6 | 19.4 | 18.1 | 18.4 | 17.8 | 19.1 | 18.6 | 17.8 |
| 17.5 | 17.3 | 16.7 | 16.6 | 16.7 | 16.6 | 16.6 | 17.5 | 17.2 | 16.8 |
| 19.7 | 19.1 | 18.4 | 17.5 | 17.5 | 18.5 | 17.7 | 17.5 | 17.7 | 17.3 |
| 17.5 | 17.3 | 18.4 | 17.6 | 16.8 | 16.5 | 16.7 | 17.1 | 16.7 | 16.5 |
| 17.0 | 16.8 | 16.0 | 15.5 | 15.9 | 16.5 | 16.5 | 16.5 | 17.1 | 16.9 |
| 16.5 | 16.6 | 16.0 | 16.5 | 16.0 | 16.0 | 15.5 | 16.5 | 15.6 | 15.8 |
| 14.9 | 14.9 | 14.5 | 14.2 | 13.0 | 13.5 | 14.5 | 15.8 | 15.0 | 16.2 |
| 15.4 | 15.6 | 16.9 | 17.4 | 17.4 | 17.1 | 17.6 | 17.1 | 16.6 | 17.0 |
| 16.8 | 15.4 | 14.8 | 16.0 | 16.8 | 17.5 | 17.1 | 16.8 | 17.0 | 16.4 |
| 16.2 | 16.2 | 17.2 | 18.5 | 20.2 | 20.8 | 20.5 | 21.5 | 22.5 | 21.9 |
| 20.1 | 20.6 | 20.0 | 20.1 | 19.4 | 19.6 | 20.5 | 19.9 | 20.0 | 19.6 |
| 19.4 | 18.8 | 18.9 | 19.6 | 20.5 | 19.4 | 19.1 | 18.2 | 18.8 | 17.9 |
| 17.5 | 17.9 | 17.9 | 17.8 | 18.1 | 18.0 | 18.0 | 19.2 | 19.1 | 20.8 |
| 19.6 | 20.4 | 21.6 | 21.2 | 21.5 | 21.1 | 21.9 | 20.8 | 16.0 | 19.9 |
| 18.8 | 18.8 | 18.1 | 18.1 | 17.8 | 17.9 | 16.9 | 17.2 | 16.9 | 16.9 |
| 16.8 | 16.9 | 16.8 | 16.4 | 16.8 | 16.9 | 16.8 | 16.5 | 16.2 | 16.2 |
| 16.0 | 15.8 | 14.6 | 14.8 | 14.5 | 13.9 | 13.9 | 14.0 | 15.0 | 14.8 |
| 14.5 | 14.8 | 15.1 | 14.9 | 15.1 | 15.0 | 16.2 | 15.8 | 16.0 | 15.9 |
| 15.8 | 17.5 | 17.9 | 17.0 | 16.8 | 16.6 | 16.6 | 16.0 | 16.2 | 15.9 |
| 14.9 | 14.8 | 14.4 | 14.2 | 14.6 | 14.1 | 14.1 | 14.4 | 14.4 | 14.1 |
| 14.2 | 14.6 | 14.5 | 14.4 | 13.8 | 13.6 | 14.4 | 13.8 | 14.1 | 13.9 |
| 14.0 | 13.9 | 13.5 | 13.8 | 12.6 | 12.6 | 12.2 | 11.4 | 12.5 | 12.1 |
| 11.6 | 11.8 | 12.0 | 12.0 | 11.8 | 11.4 | 10.8 | 10.0 | 9.9 | 9.9 |
| 9.5 | 9.5 | 10.4 | 10.8 | 10.8 | 10.1 | 11.2 | 12.1 | 11.5 | 10.8 |
| 10.8 | 10.2 | 8.9 | 8.9 | 8.1 | 7.9 | 7.9 | 9.5 | 9.0 | 8.9 |
| 9.0 | 9.1 | 9.9 | 10.4 | 10.0 | 9.2 | 9.6 | 11.2 | 13.4 | 12.4 |
| 12.2 | 11.8 | 11.1 | 10.1 | 10.2 | 10.1 | 9.0 | 9.0 | 8.6 | 9.2 |
| 8.9 | 9.5 | 9.6 | 11.0 | 10.5 | 10.8 | 10.2 | 10.1 | 9.9 | 9.6 |
| 9.2 | 9.0 | 8.9 | 9.4 | 9.5 | 10.1 | 10.0 | 9.8 | 9.0 | 9.0 |
| 9.0 | 8.6 | 8.1 | 8.2 | 7.8 | 7.1 | 7.0 | 6.8 | 6.6 | 7.2 |
| 6.2 | 7.0 | 7.2 | 7.0 | 8.0 | 8.2 | 9.0 | 9.0 | 10.0 | 10.1 |
| 9.9 | 9.9 | 9.6 | 9.5 | 9.2 | 9.8 | 10.5 | 10.4 | 11.1 | 10.8 |
| 12.5 | 12.9 | 12.4 | 13.1 | 14.4 | 13.9 | 14.0 | 13.1 | 12.9 | 13.9 |
| 14.8 | 14.9 | 13.5 | 13.5 | 13.6 | 14.8 | 14.8 | 15.1 | 15.1 | 14.5 |
| 14.0 | 13.1 | 12.9 | 11.5 | 11.2 | 11.0 | 11.5 | 11.6 | 13.4 | 13.1 |
| 13.9 | 13.9 | 13.4 | 13.0 | 13.2 | 12.9 | 13.0 | 13.9 | 12.8 | 12.8 |
| 12.2 | 11.5 | 11.4 | 11.2 | 10.5 | 11.5 | 11.2 | 12.6 | 12.0 | 11.9 |
| 12.0 | 12.2 | 13.0 | 12.8 | 13.2 | 14.2 | 15.1 | 14.4 | 13.4 | 13.5 |
| 13.1 | 12.2 | 11.8 | 11.4 | 12.0 | 13.1 | 12.6 | 12.0 | 12.6 | 12.2 |
| 12.1 | 13.0 | 12.6 | 13.1 | 13.9 | 14.1 | 13.9 | 14.4 | 15.6 | 15.1 |
| 14.8 | 14.2 | 13.8 | 13.4 | 13.0 | 13.4 | 12.6 | 12.8 | 12.8 | 12.4 |
| 12.5 | 12.2 | 11.9 | 11.8 | 11.5 | 11.1 | 11.0 | 10.8 | 10.8 | 10.9 |
| 10.8 | 10.1 | 11.5 | 11.5 | 11.0 | 10.9 | 10.8 | 10.8 | 10.1 | 9.8 |
| 9.6 | 9.6 | 8.8 | 9.4 | 8.6 | 9.1 | 8.2 | 9.2 | 8.8 | 9.0 |
| 8.9 | 9.0 | 8.9 | 10.1 | 9.8 | 10.8 | 11.4 | 11.5 | 11.4 | 12.5 |
| 11.2 | 11.1 | 10.0 | 10.0 | 9.0 | 9.4 | 8.9 | 9.2 | 9.1 | 9.8 |
| 9.5 | 8.6 | 8.6 | 9.4 | 9.6 | 9.4 | 9.2 | 9.6 | 9.9 | 10.8 |
| 11.0 | 9.2 | 9.1 | 8.9 | 8.6 | 8.2 | 7.5 | 6.6 | 8.2 | 7.1 |

ANACONDA COMPANY- ADJUSTED PRICE DATA

TIME PERIOD COVERED- FROM FEB 5,1960 TO JUNE 5,1970
WEEKLY DATA LISTED BY 10 PERIODS PER ROW
READ BY ROW- STARTING AT TOP LEFT FOR FEB 5,1960

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 29.8 | 29.8 | 29.4 | 29.1 | 25.3 | 25.8 | 26.7 | 26.0 | 25.0 | 26.8 |
| 26.5 | 25.9 | 25.1 | 25.1 | 25.0 | 25.0 | 24.6 | 25.0 | 25.7 | 25.0 |
| 24.6 | 24.1 | 24.4 | 25.6 | 24.3 | 24.9 | 24.5 | 25.7 | 25.7 | 27.0 |
| 25.5 | 24.7 | 23.4 | 22.6 | 22.4 | 22.3 | 21.2 | 21.5 | 21.6 | 22.1 |
| 23.2 | 22.1 | 22.3 | 21.7 | 22.5 | 22.5 | 22.0 | 21.8 | 23.4 | 23.5 |
| 23.8 | 24.3 | 25.1 | 24.8 | 26.0 | 26.3 | 26.6 | 25.2 | 26.4 | 26.4 |
| 27.0 | 27.3 | 29.3 | 29.8 | 30.3 | 31.3 | 30.3 | 32.3 | 31.8 | 29.5 |
| 30.1 | 28.0 | 29.2 | 28.3 | 28.6 | 27.7 | 27.5 | 28.6 | 28.3 | 27.0 |
| 27.0 | 26.8 | 27.0 | 27.3 | 26.9 | 25.6 | 25.0 | 25.0 | 25.6 | 24.5 |
| 24.0 | 24.4 | 25.0 | 25.2 | 26.0 | 25.1 | 24.6 | 24.3 | 24.2 | 24.3 |
| 25.5 | 25.3 | 25.5 | 25.1 | 25.1 | 25.1 | 25.6 | 25.1 | 24.6 | 24.4 |
| 24.6 | 23.8 | 23.5 | 23.5 | 22.9 | 23.3 | 22.0 | 22.8 | 22.3 | 22.8 |
| 21.3 | 21.6 | 20.9 | 20.5 | 19.6 | 20.1 | 20.3 | 20.9 | 20.3 | 20.3 |
| 20.1 | 20.1 | 20.5 | 20.5 | 20.5 | 19.8 | 19.0 | 18.8 | 18.5 | 18.5 |
| 18.3 | 18.0 | 18.7 | 19.3 | 20.1 | 21.8 | 22.5 | 20.9 | 21.0 | 20.6 |
| 20.3 | 20.2 | 21.6 | 21.8 | 21.5 | 22.2 | 23.6 | 22.6 | 22.6 | 22.5 |
| 22.3 | 21.9 | 22.2 | 22.5 | 22.4 | 22.9 | 23.2 | 23.5 | 23.8 | 24.2 |
| 24.3 | 25.3 | 26.1 | 26.3 | 26.7 | 25.6 | 25.3 | 25.1 | 24.8 | 24.6 |
| 23.5 | 23.6 | 23.5 | 24.1 | 25.0 | 25.0 | 25.0 | 25.2 | 25.3 | 25.4 |
| 24.6 | 25.1 | 25.0 | 25.0 | 24.8 | 24.3 | 23.9 | 23.6 | 22.5 | 22.9 |
| 22.0 | 21.3 | 21.6 | 23.0 | 24.1 | 23.7 | 24.3 | 23.5 | 22.7 | 23.1 |
| 22.8 | 24.0 | 24.1 | 24.0 | 24.8 | 24.0 | 23.8 | 21.4 | 22.0 | 21.6 |
| 21.1 | 22.3 | 22.4 | 22.0 | 21.8 | 21.4 | 20.4 | 20.1 | 21.1 | 22.0 |
| 21.8 | 22.5 | 23.0 | 22.6 | 22.5 | 22.3 | 23.3 | 22.5 | 22.5 | 23.3 |
| 24.6 | 26.9 | 26.4 | 27.0 | 27.7 | 27.1 | 28.3 | 28.3 | 28.0 | 27.3 |
| 28.0 | 28.1 | 28.0 | 27.5 | 26.6 | 26.6 | 26.1 | 28.2 | 28.3 | 29.0 |
| 30.3 | 30.0 | 28.8 | 30.5 | 31.5 | 31.0 | 32.5 | 31.4 | 30.8 | 30.5 |
| 31.1 | 32.5 | 31.8 | 32.5 | 33.1 | 33.1 | 33.6 | 34.0 | 32.6 | 31.3 |
| 32.0 | 30.3 | 31.3 | 32.0 | 31.4 | 30.3 | 33.4 | 33.8 | 31.5 | 31.5 |
| 31.7 | 32.1 | 33.8 | 34.9 | 36.1 | 36.8 | 37.5 | 38.5 | 42.7 | 42.0 |
| 39.5 | 40.5 | 40.6 | 41.0 | 40.9 | 39.8 | 41.6 | 41.0 | 40.3 | 44.0 |
| 47.0 | 47.0 | 48.5 | 46.0 | 46.5 | 46.0 | 45.5 | 43.9 | 42.7 | 41.9 |
| 44.0 | 45.1 | 48.8 | 53.3 | 49.3 | 47.3 | 44.5 | 40.8 | 42.5 | 44.9 |
| 42.6 | 42.6 | 44.5 | 43.5 | 42.2 | 43.5 | 44.6 | 42.8 | 40.6 | 38.0 |
| 38.5 | 35.1 | 33.1 | 33.8 | 36.1 | 36.3 | 36.1 | 35.5 | 34.1 | 35.8 |
| 36.8 | 39.0 | 38.7 | 39.8 | 41.1 | 41.4 | 39.0 | 40.6 | 40.0 | 40.6 |
| 40.0 | 41.6 | 44.8 | 44.2 | 45.5 | 46.5 | 45.0 | 42.8 | 45.0 | 43.4 |
| 40.8 | 42.5 | 42.3 | 41.6 | 40.9 | 41.6 | 41.7 | 44.0 | 45.3 | 44.7 |
| 45.1 | 46.0 | 46.6 | 47.0 | 49.1 | 47.8 | 47.6 | 48.5 | 47.9 | 49.2 |
| 48.1 | 52.5 | 53.0 | 51.0 | 48.6 | 50.0 | 49.4 | 50.6 | 49.0 | 48.8 |
| 48.4 | 45.8 | 46.2 | 44.9 | 44.2 | 46.0 | 44.5 | 48.2 | 45.9 | 46.6 |
| 46.8 | 46.5 | 47.4 | 46.0 | 46.2 | 45.0 | 47.5 | 46.0 | 44.5 | 43.0 |
| 39.9 | 41.1 | 42.9 | 44.9 | 42.6 | 41.5 | 41.1 | 41.5 | 43.1 | 45.8 |
| 45.1 | 45.6 | 44.5 | 49.8 | 49.8 | 52.2 | 49.5 | 51.2 | 51.2 | 54.2 |
| 51.1 | 47.9 | 45.5 | 46.2 | 46.5 | 45.0 | 46.0 | 44.2 | 47.2 | 48.0 |
| 49.0 | 49.5 | 50.1 | 50.0 | 52.4 | 53.0 | 50.5 | 52.9 | 52.8 | 52.4 |
| 57.4 | 56.0 | 60.5 | 63.9 | 60.2 | 63.9 | 64.6 | 61.2 | 61.2 | 58.2 |
| 58.5 | 57.5 | 51.9 | 52.0 | 52.0 | 51.6 | 51.8 | 52.1 | 54.9 | 55.2 |
| 52.8 | 53.0 | 53.8 | 51.1 | 47.5 | 43.0 | 43.2 | 41.1 | 40.9 | 39.9 |
| 33.8 | 32.1 | 30.2 | 30.9 | 30.4 | 30.2 | 30.2 | 28.9 | 27.9 | 29.0 |
| 28.0 | 28.2 | 28.1 | 27.8 | 27.2 | 27.9 | 29.6 | 32.0 | 30.2 | 29.6 |
| 31.0 | 29.8 | 30.1 | 28.8 | 28.1 | 28.4 | 29.8 | 30.8 | 32.0 | 29.0 |
| 28.1 | 27.5 | 27.9 | 27.8 | 28.5 | 29.5 | 28.9 | 27.6 | 28.0 | 29.4 |
| 29.2 | 29.1 | 28.0 | 27.1 | 26.9 | 26.6 | 25.0 | 24.5 | 26.5 | 25.2 |

BOEING-   ADJUSTED PRICE DATA

TIME PERIOD COVERED- FROM FEB 5,1960 TO JUNE 5,1970
WEEKLY DATA LISTED BY 10 PERIODS PER ROW
READ BY ROW- STARTING AT TOP LEFT FOR FEB 5,1960

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 14.6 | 14.5 | 14.5 | 14.0 | 13.5 | 13.5 | 13.0 | 12.1 | 12.0 | 12.1 |
| 11.9 | 12.3 | 12.6 | 12.3 | 12.1 | 14.5 | 13.6 | 13.1 | 13.7 | 14.4 |
| 13.5 | 13.5 | 13.3 | 14.3 | 14.8 | 14.6 | 17.1 | 16.0 | 16.7 | 17.0 |
| 16.2 | 15.4 | 15.0 | 14.8 | 15.3 | 15.4 | 15.8 | 15.1 | 15.6 | 16.6 |
| 17.7 | 17.3 | 17.8 | 18.3 | 19.1 | 19.3 | 18.7 | 18.4 | 19.4 | 19.9 |
| 19.5 | 18.2 | 20.1 | 19.5 | 20.8 | 20.8 | 21.0 | 21.5 | 23.2 | 22.9 |
| 22.8 | 21.6 | 21.6 | 21.8 | 22.5 | 23.3 | 23.6 | 22.6 | 23.3 | 23.1 |
| 22.3 | 23.0 | 23.0 | 24.4 | 24.6 | 25.1 | 24.6 | 26.3 | 27.0 | 26.8 |
| 27.6 | 27.3 | 27.1 | 27.7 | 26.9 | 26.5 | 26.5 | 26.0 | 26.0 | 23.3 |
| 25.0 | 24.3 | 25.0 | 23.9 | 23.5 | 24.3 | 24.6 | 26.4 | 25.8 | 25.1 |
| 24.8 | 25.8 | 26.5 | 27.1 | 27.7 | 27.8 | 27.6 | 26.1 | 26.3 | 25.6 |
| 25.3 | 25.2 | 25.1 | 24.1 | 23.8 | 24.2 | 23.2 | 23.5 | 22.1 | 22.5 |
| 20.0 | 21.8 | 21.2 | 20.4 | 18.9 | 20.4 | 20.2 | 20.7 | 20.1 | 19.8 |
| 20.5 | 19.5 | 21.0 | 21.0 | 20.3 | 19.9 | 20.6 | 19.5 | 19.0 | 19.6 |
| 19.2 | 18.5 | 18.8 | 19.6 | 21.0 | 20.9 | 21.0 | 19.5 | 19.5 | 19.2 |
| 18.5 | 18.5 | 19.3 | 19.1 | 19.5 | 19.3 | 20.0 | 19.6 | 19.3 | 18.9 |
| 19.2 | 19.0 | 18.4 | 18.1 | 18.1 | 18.8 | 18.5 | 19.1 | 18.6 | 18.9 |
| 18.6 | 18.4 | 18.6 | 18.4 | 18.3 | 17.9 | 17.8 | 17.7 | 17.8 | 17.0 |
| 16.7 | 15.9 | 16.8 | 17.2 | 16.5 | 16.9 | 17.5 | 17.7 | 17.6 | 17.1 |
| 18.3 | 17.0 | 16.8 | 17.0 | 17.3 | 17.2 | 18.1 | 18.4 | 17.3 | 19.1 |
| 19.1 | 18.2 | 18.9 | 18.3 | 18.7 | 18.6 | 18.8 | 19.1 | 19.9 | 19.9 |
| 20.1 | 20.3 | 21.1 | 21.3 | 21.3 | 21.8 | 21.8 | 23.3 | 22.9 | 23.0 |
| 23.1 | 22.1 | 23.3 | 23.6 | 25.0 | 24.6 | 24.6 | 25.1 | 26.4 | 27.1 |
| 27.5 | 28.1 | 27.1 | 27.5 | 28.0 | 28.0 | 27.1 | 28.6 | 30.0 | 32.0 |
| 31.5 | 32.3 | 32.1 | 33.1 | 33.4 | 31.9 | 31.7 | 30.6 | 32.1 | 32.5 |
| 32.9 | 34.0 | 33.6 | 34.6 | 35.3 | 34.1 | 34.5 | 34.9 | 33.9 | 32.1 |
| 33.1 | 32.6 | 33.9 | 35.1 | 34.0 | 33.6 | 32.9 | 31.5 | 30.6 | 31.7 |
| 34.6 | 36.1 | 37.1 | 37.2 | 36.8 | 37.1 | 38.1 | 36.5 | 36.0 | 34.5 |
| 34.2 | 33.3 | 34.6 | 35.5 | 36.3 | 37.1 | 38.6 | 39.3 | 40.0 | 39.0 |
| 40.8 | 42.2 | 46.3 | 46.5 | 49.0 | 50.6 | 51.3 | 52.2 | 55.8 | 61.8 |
| 61.3 | 58.5 | 66.4 | 68.0 | 67.1 | 68.8 | 69.0 | 64.3 | 65.4 | 66.9 |
| 72.8 | 75.5 | 82.3 | 84.0 | 82.8 | 84.1 | 80.5 | 78.6 | 71.8 | 70.7 |
| 77.3 | 76.5 | 82.9 | 88.3 | 78.1 | 77.3 | 71.9 | 68.8 | 65.0 | 71.6 |
| 65.9 | 69.2 | 77.9 | 74.5 | 73.4 | 72.0 | 71.2 | 66.5 | 60.4 | 62.8 |
| 60.4 | 61.1 | 50.5 | 53.8 | 53.1 | 60.4 | 57.0 | 51.0 | 47.2 | 50.0 |
| 47.0 | 48.8 | 57.1 | 60.0 | 63.0 | 67.8 | 63.0 | 62.1 | 65.0 | 64.5 |
| 65.6 | 65.8 | 66.4 | 74.1 | 71.5 | 70.0 | 71.5 | 68.0 | 71.2 | 80.6 |
| 80.9 | 82.0 | 85.6 | 82.5 | 83.0 | 86.5 | 93.0 | 95.6 | 94.2 | 98.9 |
| 97.2 | 96.1 | 95.8 | 101.0 | 106.5 | 106.9 | 104.5 | 99.6 | 101.9 | 101.0 |
| 104.0 | 108.5 | 105.1 | 100.1 | 90.2 | 97.0 | 93.2 | 92.9 | 88.8 | 86.8 |
| 84.1 | 81.5 | 87.2 | 83.2 | 85.6 | 82.1 | 86.6 | 90.2 | 91.6 | 92.0 |
| 87.2 | 88.2 | 90.5 | 83.0 | 83.5 | 78.2 | 80.9 | 77.8 | 75.0 | 77.0 |
| 78.1 | 74.9 | 73.9 | 74.2 | 70.2 | 68.5 | 68.2 | 70.4 | 72.2 | 73.9 |
| 79.8 | 76.2 | 72.6 | 71.4 | 69.5 | 71.8 | 67.1 | 67.4 | 63.5 | 63.2 |
| 67.1 | 66.2 | 62.4 | 58.2 | 58.5 | 56.4 | 53.8 | 57.2 | 56.8 | 56.8 |
| 55.0 | 54.8 | 61.2 | 58.6 | 56.9 | 57.8 | 55.0 | 53.9 | 55.2 | 57.1 |
| 59.4 | 58.4 | 56.9 | 57.4 | 56.0 | 56.5 | 55.8 | 56.9 | 58.0 | 59.0 |
| 59.4 | 59.5 | 55.6 | 52.6 | 50.1 | 48.4 | 49.5 | 50.9 | 48.8 | 50.2 |
| 50.0 | 48.5 | 45.0 | 46.8 | 45.6 | 45.1 | 44.0 | 43.1 | 42.6 | 41.1 |
| 41.9 | 41.0 | 36.9 | 36.6 | 35.8 | 34.1 | 31.2 | 30.6 | 31.6 | 37.4 |
| 34.9 | 33.8 | 33.5 | 34.2 | 32.5 | 32.8 | 33.2 | 32.2 | 31.8 | 31.8 |
| 33.2 | 31.5 | 31.1 | 30.1 | 28.6 | 28.5 | 29.5 | 29.8 | 30.2 | 26.8 |
| 26.5 | 23.0 | 21.4 | 20.0 | 24.5 | 25.0 | 23.2 | 22.8 | 21.5 | 23.8 |
| 23.1 | 22.1 | 20.9 | 20.2 | 20.2 | 19.9 | 20.0 | 17.8 | 18.2 | 15.8 |

UNITED AIR LINES- ADJUSTED PRICE DATA

TIME PERIOD COVERED- FROM FEB 5,1960 TO JUNE 5,1970
WEEKLY DATA LISTED BY 10 PERIODS PER ROW
READ BY ROW- STARTING AT TOP LEFT FOR FEB 5,1960

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 12.1 | 12.9 | 13.0 | 12.9 | 11.9 | 11.7 | 11.7 | 11.4 | 10.9 | 12.4 |
| 12.1 | 11.7 | 12.2 | 13.8 | 13.0 | 13.4 | 13.4 | 12.9 | 13.8 | 12.3 |
| 13.3 | 12.9 | 13.0 | 12.8 | 11.5 | 13.1 | 13.1 | 13.8 | 13.4 | 14.1 |
| 13.4 | 13.9 | 12.9 | 12.2 | 12.6 | 12.4 | 13.6 | 13.2 | 12.9 | 14.2 |
| 13.9 | 13.7 | 13.4 | 13.9 | 15.2 | 14.9 | 14.5 | 15.8 | 16.1 | 17.0 |
| 16.8 | 17.3 | 17.6 | 18.7 | 18.8 | 18.6 | 18.2 | 18.3 | 18.2 | 17.8 |
| 18.4 | 20.4 | 20.3 | 21.0 | 20.5 | 21.4 | 22.7 | 21.9 | 21.4 | 21.3 |
| 21.4 | 20.0 | 20.1 | 19.9 | 21.6 | 21.4 | 20.9 | 20.9 | 21.9 | 21.5 |
| 21.0 | 19.5 | 19.8 | 19.4 | 18.6 | 18.5 | 19.0 | 18.6 | 17.7 | 15.8 |
| 16.4 | 16.5 | 16.8 | 17.3 | 16.5 | 16.3 | 17.0 | 17.1 | 17.3 | 17.8 |
| 17.4 | 17.5 | 18.4 | 17.3 | 17.7 | 17.4 | 17.2 | 16.6 | 17.2 | 16.6 |
| 16.6 | 16.2 | 16.3 | 15.4 | 15.5 | 15.7 | 15.0 | 14.9 | 14.1 | 14.3 |
| 14.0 | 13.0 | 12.8 | 11.2 | 10.0 | 11.1 | 12.0 | 11.6 | 12.1 | 12.3 |
| 14.2 | 14.3 | 14.6 | 14.9 | 15.3 | 15.1 | 15.8 | 14.0 | 13.2 | 13.3 |
| 13.6 | 12.7 | 12.0 | 13.2 | 13.7 | 14.0 | 14.6 | 15.3 | 15.8 | 15.1 |
| 15.3 | 15.2 | 15.8 | 16.4 | 16.3 | 16.3 | 17.2 | 16.8 | 17.3 | 16.9 |
| 16.1 | 16.3 | 16.5 | 17.7 | 17.8 | 19.2 | 19.3 | 19.4 | 19.4 | 19.8 |
| 19.5 | 20.8 | 21.0 | 20.6 | 20.7 | 20.9 | 21.5 | 19.0 | 19.3 | 18.8 |
| 18.4 | 19.0 | 19.6 | 19.8 | 20.0 | 19.6 | 20.0 | 20.5 | 19.2 | 18.5 |
| 18.5 | 13.1 | 18.5 | 18.1 | 18.2 | 18.4 | 18.8 | 20.3 | 17.3 | 20.3 |
| 20.6 | 20.8 | 20.7 | 21.5 | 22.5 | 23.8 | 23.5 | 24.6 | 23.8 | 25.3 |
| 26.2 | 27.8 | 27.6 | 28.0 | 27.9 | 29.7 | 30.5 | 32.1 | 30.6 | 30.0 |
| 28.1 | 28.8 | 28.3 | 28.1 | 28.6 | 27.4 | 25.3 | 25.7 | 26.0 | 27.1 |
| 27.1 | 26.8 | 26.1 | 26.0 | 25.7 | 24.6 | 25.0 | 24.5 | 25.3 | 24.1 |
| 24.1 | 22.9 | 24.4 | 25.1 | 26.3 | 26.7 | 26.8 | 27.1 | 27.4 | 29.3 |
| 30.0 | 29.3 | 30.0 | 29.8 | 30.5 | 30.2 | 29.6 | 31.2 | 32.6 | 32.6 |
| 30.5 | 31.7 | 31.8 | 34.0 | 34.8 | 35.3 | 36.0 | 34.8 | 33.5 | 34.5 |
| 36.1 | 35.9 | 37.4 | 36.7 | 37.8 | 39.1 | 37.0 | 39.1 | 38.8 | 37.1 |
| 35.8 | 34.1 | 35.8 | 37.3 | 37.5 | 36.0 | 34.6 | 35.5 | 36.8 | 36.1 |
| 38.3 | 40.1 | 40.5 | 42.4 | 44.1 | 43.5 | 43.7 | 47.1 | 47.0 | 49.1 |
| 50.4 | 48.0 | 47.6 | 51.1 | 54.6 | 58.1 | 58.8 | 53.7 | 52.4 | 50.5 |
| 53.3 | 54.3 | 57.9 | 58.0 | 59.9 | 59.5 | 58.9 | 58.9 | 58.3 | 59.0 |
| 65.8 | 69.1 | 74.0 | 73.6 | 68.0 | 68.8 | 63.9 | 61.9 | 61.0 | 61.5 |
| 62.0 | 68.4 | 71.4 | 69.0 | 67.2 | 68.5 | 68.9 | 63.8 | 64.2 | 61.8 |
| 61.5 | 58.6 | 47.9 | 46.6 | 47.8 | 53.0 | 51.0 | 51.0 | 44.4 | 48.5 |
| 46.5 | 45.9 | 49.5 | 51.2 | 52.0 | 56.8 | 60.5 | 62.5 | 62.9 | 62.8 |
| 61.0 | 61.5 | 67.5 | 70.1 | 67.1 | 70.0 | 68.0 | 65.5 | 67.5 | 71.6 |
| 77.6 | 79.6 | 79.9 | 76.5 | 77.0 | 80.1 | 82.1 | 82.8 | 82.9 | 86.0 |
| 80.5 | 82.0 | 77.0 | 83.6 | 79.0 | 75.9 | 72.4 | 78.0 | 81.2 | 79.6 |
| 82.1 | 80.6 | 78.6 | 79.9 | 77.5 | 76.0 | 70.5 | 71.5 | 69.4 | 68.9 |
| 65.1 | 60.8 | 58.0 | 61.4 | 57.2 | 58.1 | 63.8 | 67.0 | 63.4 | 58.8 |
| 61.6 | 63.9 | 66.0 | 59.8 | 60.2 | 57.0 | 54.2 | 50.0 | 48.6 | 49.0 |
| 49.6 | 44.2 | 43.6 | 43.9 | 40.4 | 41.0 | 46.5 | 49.8 | 43.1 | 42.9 |
| 42.5 | 42.6 | 41.2 | 40.9 | 40.5 | 43.1 | 41.6 | 40.2 | 40.6 | 40.6 |
| 42.8 | 41.9 | 37.8 | 37.2 | 35.5 | 35.6 | 36.8 | 35.4 | 37.5 | 41.5 |
| 43.5 | 41.6 | 42.9 | 44.0 | 45.8 | 44.8 | 43.0 | 44.6 | 44.5 | 44.1 |
| 44.9 | 42.2 | 40.8 | 43.9 | 44.4 | 41.9 | 42.0 | 47.0 | 46.4 | 47.4 |
| 45.8 | 46.0 | 42.4 | 42.8 | 40.5 | 38.0 | 39.8 | 37.6 | 36.9 | 39.6 |
| 39.5 | 40.8 | 40.6 | 37.1 | 38.0 | 36.5 | 38.8 | 37.5 | 38.0 | 39.6 |
| 36.4 | 37.5 | 33.4 | 31.8 | 27.9 | 30.4 | 29.0 | 28.8 | 32.9 | 30.2 |
| 28.1 | 30.0 | 31.0 | 28.5 | 28.6 | 27.8 | 32.6 | 32.9 | 33.6 | 35.6 |
| 32.5 | 32.5 | 31.9 | 29.2 | 29.0 | 27.5 | 29.5 | 28.0 | 27.6 | 24.4 |
| 23.9 | 22.1 | 22.1 | 24.8 | 24.2 | 25.4 | 25.1 | 23.4 | 23.6 | 23.8 |
| 23.2 | 23.1 | 22.4 | 22.0 | 20.4 | 20.2 | 18.5 | 17.0 | 18.2 | 15.0 |

APPENDIX C

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C      ADJUSTED EXSMO PROGRAM FOR OPTIMAL SMOOTHING CONSTANT
C      AND SMALLEST FORECAST ERROR FOR T FORECAST PERIODS
C
C      PURPOSE-READ THE PROBLEM PARAMETER CARD AND A TIME SERIES
C              - CALL SUBROUTINE EXSMO
C              - PRINT SMOTHED RESULTS
C
C      REMARKS - ASMOOTHING CONSTANT SPECIFIED IN THE PROBLEM PARAMETER CARD
C             MUST BE GREATER THAN ZERO AND LESS THAN ONE TO OBTAIN REASONABLE
C             RESULTS.
C             - SUBROUTINE EXSMO REQUIRED
C
C      FOOTNOTE- BROWN. R.G., SMOOTHING, FORECASTING, AND PREDICTION OF DISCRETE
C              TIME SERIES,PRENTICE-HALL, N.Y., 1963, PP. 140-44.
C
C      REQUIREMENTS - DIMENTION GREATER OR EQUAL TO NUMBER OF DATA POINTS.
C
       DIMENSION X(600),S(600),SDIF(600)
   101 FORMAT(A6,I2,I4,F5.3,3F10.0)
   102 FORMAT(10(F7.1,1X))
   103 FORMAT(34HTRIPLE EXPONENTIAL SMOOTHING...    ,A6,I2//22HNUMBER OF
      1DATA POINTS,I4/)
   104 FORMAT(13HCOEFFICIENTS ,9X,1HA,14X,1HB,14X,1HC)
   105 FORMAT(8HORIGINAL,F19.5,2F15.5)
   106 FORMAT(8HUPDATED ,4X,3E15.8/)
   107 FORMAT(28X,13HSMOOTHED DATA/8X,10HINPUT DATA,8X,10H(FORECAST),
      18X,10HDIFFERENCE,5X,12HDIFF SQUARED)
   108 FORMAT(5X,F13.5,5X,F13.5,5X,F13.5,4X,E15.8)
   109 FORMAT(31X,8HTOTALS--,E15.8,4X,E15.8/)
   110 FORMAT(10X,31HTHE MEAN OF THE DIFFERENCES IS ,E15.8//)
   111 FORMAT(17X,39HTHE VARIANCE OF THE FORECAST ERRORS IS ,E15.8//)
   112 FORMAT(6X,50HTHE DEVIATION OF THE FORECAST ABOUT THE ACTUAL IS ,
      1E15.8//)
   113 FORMAT(2F5.3,2F2.0)
   115 FORMAT(28X,23HSMOOTHING CONSTANT IS  ,F5.3/)
   118 FORMAT(31X,20HFORECAST PERIOD IS  ,F4.0/)
C
     1 READ 101,PR,PRI,NX,AL,A,B,C
C
C      TABLE
C      PR= PROBLEM NAME
C      PRI = PROBLEM NUMBER
C      NX = NUMBER OF DATA POINTS
C      AL=SMOOTHING CONSTANT-READ IN ONE INCREMENT LESS THAN INITIAL CONSTANT
C      A,B,C = COEFFICIENTS OF THE PREDICTION EQUATION
C      ALINC=SMOOTHING CONSTANT INCREMENT CHANGE DESIRED
C      ALTERM=FINAL VALUE OF SMOOTHING CONSTANT DESIRED
C      TI=ONE PERIOD LESS THAN INITIAL FORECAST PERIOD DESIRED
C      TFIN==LAST FORECAST PERIOD DESIRED
C      MUST READ IN AL, ALINC, ALTERM, TI, AND TFIN
C
       PUNCH 103,PR,PRI,NX
       PUNCH 104
       PUNCH 105,A,B,C
       READ 102,(X(I),I=1,NX)
       READ 113,ALINC,ALTERM,TI,TFIN
     5 AL = AL + ALINC
       AAL = AL
       T = TI
     6 T = T + 1.0
       CALL EXSMO(X,NX,AL,A,B,C,S,SDIF,SQDIF,SUMDF,T)
       PUNCH 106,A,B,C
       PUNCH 115,AAL
       PUNCH 118,T
       IF(SENSE SWITCH 4)4,7
     4 PUNCH 107
       DO 2 I=101,NX
       XX = SDIF(I)**2
       PUNCH 108,X(I),S(I),SDIF(I),XX
     2 CONTINUE
```

```
    7 PUNCH 109,SUMDF,SQDIF
      ANX = NX-100
      XMEAN = SUMDF/ANX
      VARXX = (SQDIF/(ANX-1.0))
      STDEV = (VARXX)**.5
      PUNCH 110,XMEAN
      PUNCH 111,VARXX
      PUNCH 112,STDEV
      IF(T-TFIN)6,8,8
    8 CONTINUE
C
      IF(AL-ALTERM)5,3,3
    3 GO TO 1
      END
C
C     EXSMO SUBROUTINE
C
C     TRIPLE EXPONENTIAL SMOOTHED SERIES OF THE GIVEN SERIES X.
C
C     PARAMETERS
C     X = INPUT VECTOR OF LENGTH NX CONTAINING TIME SERIES DATA
C     NX = ELEMENTS OF X
C     AL = SMOOTHING CONSTANT ALPHA GREATER THAN ZERO LESS THAN ONE
C     A = COEFFICIENTS OF THE PREDICTION EQUATION
C     B   WHERE S IS PREDICTED
C     C   T PERIODS AHEAD BY A+B*T + C*T*T/2, AS INPUT... IF A=B=C=0,
C     PROGRAM WILL PROVIDE INITIAL VALUES.  IF AT LEAST ONE OF A,B,C IS
C     NOT ZERO, PROGRAM WILL TAKE GIVEN VALUES AS INITIAL VALUES.
C     AS OUTPUT- A,B,C CONTAIN LATEST, UPDATED COEFFICIENT OF PREDICTION.
C     S = OUTPUT VECTOR OF LENGTH NX CONTAINING TRIPLE EXPONENTIALLY
C     SMOOTHED TIME SERIES.
C
      SUBROUTINE EXSMO(X,NX,AL,A,B,C,S,SDIF,SQDIF,SUMDF,T)
C
      DIMENSION X(600),S(600),SDIF(600)
      IF(A) 40,10,40
   10 IF(B) 40,20,40
   20 IF(C) 40,30,40
   30 C=X(1)-2.0*X(2)+X(3)
      B=X(2)-X(1)-1.5*C
      A=X(1)-B-0.5*C
   40 BE=1.0-AL
      BECUB=BE*BE*BE
      ALCUP=AL*AL*AL
      SQDIF=0.0
      SUMDF=0.0
      J = T
      DO 42 K=1,J
   42 S(K) = X(K)
      NXX = NX+J-1
      DO 50 I=J,NXX
C     FIND S(I) FOR T PERIODS AHEAD
      S(I)=A+B*T+0.5*C*T*T
C
C     UPDATE COEFFICIENTS A,B,C
      N=I-J+1
      DIF=S(N)-X(N)
      SDIF(N)=DIF
      IF(100-N)45,48,48
   45 SQDIF=SQDIF+DIF**2
      SUMDF=SUMDF+DIF
   48 A=X(N)+BECUB*DIF
      B=B+C-1.5*AL*AL*(2.0-AL)*DIF
   50 C=C-ALCUP*DIF
C
      RETURN
      END
```